MCS 152(Summer)MT1 Questions

Q1. Evaluate the double integrals;

a)$\int_0^1 \int_0^{x^2} (x+2y)dydx$.(*Ans.* $\frac{9}{20}$.)

b)$\int_R \int e^{-x^2-y^2} dA$, where R is the region bounded by the semicircle $x = \sqrt{4-y^2}$ and the y- axis.(*Ans.* $\pi(1-e^{-4})$)

Q2. Combine the following sum into one double integral and evaluate it.

$$\int_{\frac{1}{\sqrt{2}}}^{1} \int_{\sqrt{1-x^2}}^{x} dydx + \int_{\sqrt{2}}^{2} \int_0^{\sqrt{4-x^2}} dydx + \int_1^{\sqrt{2}} \int_0^{x} dydx$$

Q3. Evaluate

$$\int_0^2 \int_x^{4-x} (x+y)e^{y-x} dxdy \text{ using substitution } u = x+y, v = x-y.$$

Q4. Reverse the order

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \sin(e^{y^3+x^2})dydx.$$

Q5. Convert to polar coordinates and evaluate

$$\int_0^1 \int_{\sqrt{1-y^2}}^{\sqrt{4-y^2}} \sqrt{y^2+x^2}dxdy + \int_1^2 \int_0^{\sqrt{4-y^2}} \sqrt{y^2+x^2}dxdy.$$

Q6. Evaluate

$$\int_0^1 \int_{1-x}^{\sqrt{1-x^2}} \frac{dydx}{(y^2+x^2)^{\frac{3}{2}}}$$

Q7. Using a suitable substitution, evaluate

$$\int_R \int (3x+6y)^2 dA$$

over the region bounded by the lines $x-2y=-2, x+2y=2, x+2y=-2, x-2y=2$.

Q8. Evaluate the integral

$$\int_0^4 \int_{\sqrt{x}}^2 \sqrt[3]{1+y^3}dydx$$

Q9. Sketch the region and write an equivalent integral with the order of integration reversed. Then evaluate both integral to confirm their equality:

a) $\int_0^1 \int_2^{4-2x} dydx$.(*Ans.* 1.)

b) $\int_0^1 \int_1^{e^x} dydx$.(*Ans.* $e-2$.)

c) $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3ydxdy$.(*Ans.* 2.)

Q10. Evaluate the integrals

a) $\int_0^1 \int_0^{x^3} e^{\frac{y}{x}} dydx$

b) $\int_0^1 \int_{\sqrt[3]{y}}^1 \frac{2\pi \sin(\pi x^2)}{x^2} dxdy$

c) $\int_0^1 \int_{2y}^2 \cos(x^2) dxdy$

Q11. Reverse the order $\int_{-1}^1 \int_{x^3}^{x^2} f(x,y)dydx$.

Q12. Change the order of the integration $\int_0^1 \int_{x^2-1}^{x} f(x,y)dydx$.

Q13. Consider the region R bounded by the curves; $y = x, 3y = x, xy = 2, xy = 4$. Let T be the transformation $\{u = \frac{x}{y}, v = xy\}$. Evaluate $\int_R \int \frac{x}{y} dxdy$.

Q14. Using Lagrange Multiplier's Method, find the maximum (minimum) values that $f(x,y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

Q15. Find the local maximum, local minimum and saddle points of the function $f(x,y) = (x-1)(y+1)(x+y-3)$.